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Abstract

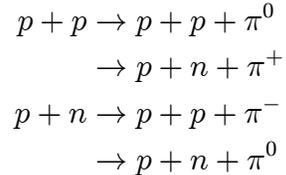
A new “Generalized” Weizsäcker-Williams Method (GWWM) is used to calculate approximate cross sections for relativistic peripheral proton-proton collisions. Instead of a massless photon mediator, the method allows for the mediator to have mass for short range interactions. This method generalizes the Weizsäcker-Williams Method (WWM) from Coulomb interactions to GWWM for strong interactions. An elastic proton-proton cross section is calculated using GWWM with experimental data for the elastic π^+p interaction where the massive π^+ is now the mediator. The resulting calculated cross section is compared with existing data for the elastic proton-proton interaction. A good approximate fit is found between the data and calculation.

1. Introduction

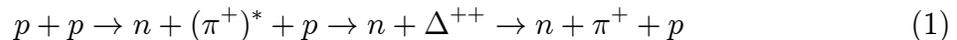
The focus of this project is a very specific type of reaction—pion production in proton-proton collisions. The pion-nucleon interaction is interesting for a number of reasons. First, pions live long enough ($\approx 10^{-8}$ seconds) that intense pion beams can be prepared in the laboratory and these pion-nucleon interactions can be studied in detail (ref. 1). Second, the pion is the lightest of all mesons, being more than three times lighter than the next heavier one, which has the consequence that below a certain energy threshold (500 MeV), pion-nucleon interactions can be studied without interference from other mesons. Third, because it is the lightest of the mesons, it has the greatest range according to the usual range-mass relation—*range* $\approx 1/mass$. It alone can account for the long-range part of the nuclear force, which allows for easy comparisons of experimental data with theoretical predictions without complications due to other mesons. Lastly, it is found that roughly 80 to 90 percent of all particles produced in nuclear scattering experiments are pions; the rest are kaons, baryons, antibaryons, and other particles (ref. 2). This fact has far reaching consequences for the NASA radiation problem, which is concerned with protecting inhabitants in the space environment from exposure to space radiation. There are three major sources of such radiation: galactic cosmic radiation, solar energetic particles, and anomalous cosmic radiation. All have atomic nuclei as basic constituents. Pions have the greatest multiplicity of all particles produced when space radiation impinges on a spacecraft. Because pions are very copiously produced in such interactions, the understanding of pion production in proton-proton collisions is important. The aim of this work is thus to develop an effective (semiempirical) theory that can accurately predict cross sections for this reaction.

2. Pion Production

Pions are most readily produced (in meson factories, for example) from collisions of protons with nuclear targets (ref. 3). The simplest nucleon-nucleon reactions are



Of the two proton-proton reactions listed, the latter has the greater cross section by a factor of about 5 (ref. 3). It proceeds by way of virtual pion (π^+)* exchange (the strong force). The π^+p cross section is dominated by the Delta resonance at a center of mass energy of 1232 MeV (refs. 3 and 4). The same resonance also occurs in elastic π^-p scattering, inelastic nucleon-nucleon scattering, inelastic e^-p scattering, and photoproduction. The cross sections for these reactions have other weaker resonances as well. Four of these resonances are interrelated charge states (+2, +1, 0, and -1); they are collectively called Delta resonances (Δ^{++} , Δ^+ , Δ^0 , and Δ^- , respectively). The dominant proton-proton pion production mechanism is thus



That is, two protons approach one another, and a *virtual* pion is emitted by one proton, thereby turning it into a neutron; the virtual pion and the other proton then combine to form a Δ^{++} resonance; finally (some 6×10^{-24} seconds later), the Δ^{++} decays into a *real* pion and a proton (ref. 1).

3. Generalized Weizsäcker-Williams Method

The effective theory developed in this project finds inspiration in a semiclassical theory that was first introduced in 1924 by E. Fermi and was extended 10 years later by C. Weizsäcker and E. Williams. It is now called the Weizsäcker-Williams Method (WWM), or Equivalent Photon Approximation (EPA) (refs. 5 and 6). The method aims to simplify the analysis of the collision between two charged particles. Its most basic assumption is that the colliding particles are traveling at very nearly the speed of light. Due to relativistic effects, the electric \mathbf{E} and magnetic \mathbf{B} fields of such a particle are (Lorentz) contracted into the plane that is transverse to the direction of motion. At every point in this plane, the \mathbf{E} and \mathbf{B} fields are of very nearly the same magnitude and are transverse to one another, very much like on the wavefront of a plane electromagnetic (EM) wave. In fact, to an observer (viz, another particle) at rest some short distance away from the passing particle, the effects of these fields are practically indistinguishable from those of

a passing EM wave. If the particle’s EM fields are approximated as EM plane waves, the problem of analyzing a peripheral (noncontact) collision of two ultrarelativistic charged particles thus simplifies to one of analyzing the interaction between a passing EM wave and just one particle. Besides this first assumption, that (1) the colliding particles are ultrarelativistic, there are only a few other basic approximations made; (2) the particles follow straight-line trajectories; (3) the particles have no internal structure; and (4) the photons mediating the interaction are on-shell (i.e., massless). The method can be refined by, for example, relaxing approximations (2) and (3), but in its simplest form, the WWM turns out to be an impressively accurate approximation scheme. The effective theory developed here is a refinement, or, rather, a generalization, of the WWM based on the relaxation of approximation (4). The goal is to develop a version of the method that can be used to analyze peripheral collisions mediated by *massive* mediators. Such “heavy photons” might be any of the pions mediating strong interactions or the W and Z bosons mediating electroweak interactions. The hope is that this Generalized WWM (GWWM) will turn out to be as accurate for strong and weak interactions as the original version is for EM interactions.

There are usually two parts to the typical WWM analysis—a semiclassical part and a quantum part. The semiclassical part involves the emission of EM energy (a photon in the quantum viewpoint) from an incident particle. The important quantity here is the number spectrum $N(E)$ of photons—the differential number $dn(E)$ of photons of energy E in the particle’s EM fields per unit photon energy dE :

$$N(E) = \frac{dn(E)}{dE} = \frac{\text{Differential number of photons of energy } E}{\text{Differential photon energy}} \quad (2)$$

Usually, the quantum part involves the description of the interaction between the emitted photon and a target particle. The quantity of interest here is the (microscopic) cross section σ_{mic} for this photon-induced subprocess. The determination of σ_{mic} proceeds by way of either the Feynman rules or experimental data, if available. The (macroscopic) cross section σ_{mac} for the overall process is found by folding $N(E)$ with $\sigma_{mic}(E)$:

$$\sigma_{mac} = \int dE N(E) \sigma_{mic}(E) \quad (3)$$

where the integral runs over all allowable photon energies.

The mediator for the reaction of interest in this paper is a massive pion instead of a photon. Since the microscopic π^+p cross sections are readily accessible, the problem is therefore to generalize the scope of the familiar WWM number spectrum function $N(E)$ to be able to accommodate massive pions as well as the usual massless photons as mediators. Once a generalized $N(E)$ function is devised, the overall macroscopic cross section for this interaction of interest can be immediately calculated (by way of eq. (3)).

The derivation of the generalized number spectrum function $N(E)$ proceeds in exactly the same way as outlined in references 5 or 7 for the traditional version of the WWM. The only difference is the form of the electric \mathbf{E} and magnetic \mathbf{B} fields, and the scalar Φ and vector \mathbf{A} potentials. It can be shown that the 4-potential $A^\mu \equiv (\Phi, \mathbf{A})$ of a relativistic charged particle is generally of the form

$$A^\mu = \frac{Qu^\mu}{r} e^{-mr} \quad (4)$$

Gaussian units (where $\varepsilon_0 = \mu_0 = 1$) with $\hbar = c = 1$ are being used here, and the metric is taken to be $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$. Equation (4) is actually only correct at ultrarelativistic energies; at lower energies, it is a bit more complicated. Q is an effective “charge” of the particle that is unique to each mediator; $u^\mu = \gamma(1, \mathbf{v})$ is the 4-velocity of the particle in the observer’s rest frame where $\gamma = 1/\sqrt{1-v^2}$ is the usual Lorentz factor; r is defined as $r \equiv \sqrt{b^2 + (\gamma vt)^2}$ where b is the impact parameter of the collision and t is the observer’s time coordinate; and, finally (and more importantly), m is the mass of the mediator. Note that when m is set to 0 (and Q is set to the electric charge of the particle), this equation simplifies to the usual 4-potential encountered in electrodynamics. The “massive photon” \mathbf{E} and \mathbf{B} fields are related to these Φ and \mathbf{A} potentials in the same way that they are in massless-photon electrodynamics:

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t} \quad (5a)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (5b)$$

After following a parallel route to $N(E)$, the following generalized number spectrum functions are found:

$$N_T(E) = \frac{N_0}{E} \left\{ \chi K_0(\chi) K_1(\chi) - \frac{1}{2} v^2 \chi^2 [K_1^2(\chi) - K_0^2(\chi)] \right\} \quad (6a)$$

$$N_L(E) = \frac{N_0}{E} \left\{ \frac{1}{2} (mb_{min})^2 [K_1^2(\chi) - K_0^2(\chi)] \right\} \quad (6b)$$

where

$$N_0 \equiv \frac{2}{\pi} \frac{q^2}{v^2} = \text{const} \quad (7)$$

and

$$\chi \equiv b_{min} \sqrt{m^2 + \left(\frac{E}{\gamma v}\right)^2} \quad (8)$$

The functions K_0 and K_1 appearing in equations (6a) and (6b) are modified Bessel functions of the second kind, of orders 1 and 2, respectively. Another effective charge, q , is related to Q in a not-so-trivial way. It is calculable if the mediators are any of the

bosons of the electroweak theory. For the case at hand, where the exact pion-proton coupling is unknown, q is correspondingly completely unknown and is to be regarded as a free parameter. The minimum impact parameter of the collision, b_{min} , is always a free parameter of any version of the WWM that is formulated in position-space. This quantity represents the distance of closest approach between the colliding particles before the collision can no longer be termed peripheral. For nuclei, b_{min} can usually be simply set to the nuclear radius, which is known to be $R \simeq 1.25 A^{1/3}$ fm where A is the atomic mass number of the nucleus (ref. 3).

Another distance scale that must not be overlooked is the Compton wavelength of the boson, which is the scale within which the exact location of the boson is completely uncertain. The Compton wavelength is given as λ/E , where λ is another free parameter of the theory that should be ≈ 1 . In practice, b_{min} is taken to be the greater of λ/E and $1.25 A^{1/3}$ fm as it is the larger of these two quantities that sets this minimum length scale. Finally, there is the question of the choice of boson mass m . The mediator of interest is the π^+ , which has a mass of 139.57 MeV (ref. 8). Simply plugging in this value for m yields cross sections that are in surprisingly good agreement with experimental data. It can be argued, though, that this assignment is only valid so long as the mediator is lighter than the parent particles; note that the pion is lighter than a proton. If the mediator were a W or Z boson, with a mass far exceeding that of the parent particle, a more careful analysis would be needed as energy would be violated if the familiar on-shell value ($m_W = 80.419$ GeV and $m_Z = 91.1882$ GeV) were used in that case (ref. 8).

Besides the issue of what exact values to use for q and λ , there is the complication of the boson helicity state. A boson in a transverse T helicity state has its polarization vector oriented in the plane transverse to its direction of motion, while a longitudinal L boson has its polarization vector aligned either parallel or antiparallel to its direction of motion. Alternatively, the angular momentum vector of a T boson is either parallel to or antiparallel to its direction of motion and that of an L boson is perpendicular to its direction of motion. The GWWM assumes that the boson is exclusively in either the T state or the L state, so the number spectra corresponding to the types of bosons (eq. (6a) for T bosons and eq. (6b) for L bosons) are independent. Massless photons have spin, $S = \hbar$, and are always found only in T states, which is completely consistent with these equations—when m is set to 0, N_L vanishes, and N_T reduces to the familiar formula in the traditional version of the WWM (ref. 5). Pions, on the other hand, have spin, $S = 0$, which means they transport zero angular momentum along with them. The appropriate choice of generalized number spectra to be used to represent the swarm of pions around the particle must reflect this fact. It seems reasonable to take the sum $N_T(E) + N_L(E)$ as this function, as each count represents the *number* of bosons in a *particular helicity state*. The sum of those two functions thus represents the *total number* of all bosons, in *all helicity states*, and so describes a spectrum of unpolarized bosons. The agreement between the results of this paper and experimental data shows that this assumption is a reasonable one to make.

4. Application of GWWM

The reaction of interest to this study is shown in equation (1). The equation for the cross section is a variation of equation (3). The appropriate revision reads

$$\sigma_{mac} = \int dE [N_T(E) + N_L(E)] \sigma_{mic}(E) \quad (9)$$

In this effective theory, the variable E represents the energy of the virtual pion mediating the reaction. The cross section, $\sigma_{mic}(E)$, for elastic π^+p scattering is obtained from reference 8 (in the form of experimental data). The exact set of data used for this functional is plotted in figure 37.21 on page 235 in reference 8 and can be obtained from the following URL: http://pdg.lbl.gov/~sbl/pipp_elastic.dat. The upper and lower limits (on E) of integration are naturally built into this data set. The raw data were a table of cross sections versus π^+ momenta in the lab frame. While error bars were included in the original data set, they were not incorporated into the crude calculations of this project. In order to correctly implement these data, the values of π^+ momenta in the lab frame were transformed within a Fortran code into π^+ center of mass energy values. These data were then folded with $N_T(E) + N_L(E)$ to calculate the macroscopic cross section. As mentioned previously, the mass of the pion was assumed to be the on-shell value, 139.57 MeV, and b_{min} was taken to be the greater of λ/E (where $\lambda \approx 1$) and $1.25 A^{1/3}$ fm. The final results (macroscopic cross section values) were then compared with data for elastic proton-proton scattering (with error bars); that is, for the reaction $p + p \rightarrow p + p$. That set of data is plotted in figure 37.19 on page 233 in reference 8 and can be obtained from the following URL: http://pdg.lbl.gov/~sbl/pp_elastic.dat. The $p + p \rightarrow p + p$ reaction is, of course, not the correct one to which results should be compared, but accurate data for the reaction of interest, $p + p \rightarrow n + p + \pi^+$, were not available. Reference 3 shows a plot of cross sections versus proton laboratory energies for this reaction but does not provide the actual data. The cross sections that can be read off the plot are very close in value to those for the elastic $p + p \rightarrow p + p$ reaction found in reference 8. Data from this other reaction were used as a ballpark estimate for the desired cross section data (for the $p + p \rightarrow n + p + \pi^+$ reaction).

The results of this study are shown in figure 1 using only the transverse number spectrum $N_T(E)$ and in figure 2 where both the transverse and longitudinal number spectra were used $[N_T(E) + N_L(E)]$. The cross section for the reaction, in mb, is plotted along the vertical axis and the momentum of the proton beam in the lab frame, in GeV, is plotted along the horizontal axis. As can clearly be seen, there is excellent agreement between the effective theory developed here and experimental data. Although, once again, the data shown in this plot (fig. 2) correspond to elastic $p + p \rightarrow p + p$ reactions and not $p + p \rightarrow p + n + \pi^+$ reactions. Of the two free parameters of the theory, q and λ , only adjustments in the value of q have any obvious direct effect on the predicted cross

sections. Because σ_{mac} varies with λ only logarithmically (roughly), small variations in the value of λ do not have appreciable effects. The best “eyeball” fit of the GWWM curve to the data was achieved with the values $q = 3.2$ and $\lambda = 1$ for figure 1 and $q = 2.58$ and $\lambda = 1$ for figure 2. For comparison, if q were representing the *electric charge* of the proton, it would have the value $q = e = \sqrt{\alpha} = 0.08542 \simeq 1/11.7$ where $\alpha \simeq 1/137$ is the fine-structure constant (ref. 8). The success of using the GWWM as an effective theory of strong interactions provides some motivation for further refinements of the approximation scheme. Two possible directions for future work are: (1) to obtain a data set that accurately describes the actual reaction of interest, $p + p \rightarrow n + p + \pi^+$, and data sets for other strong interaction processes as well, and (2) to try to explain why q has the value that it does.

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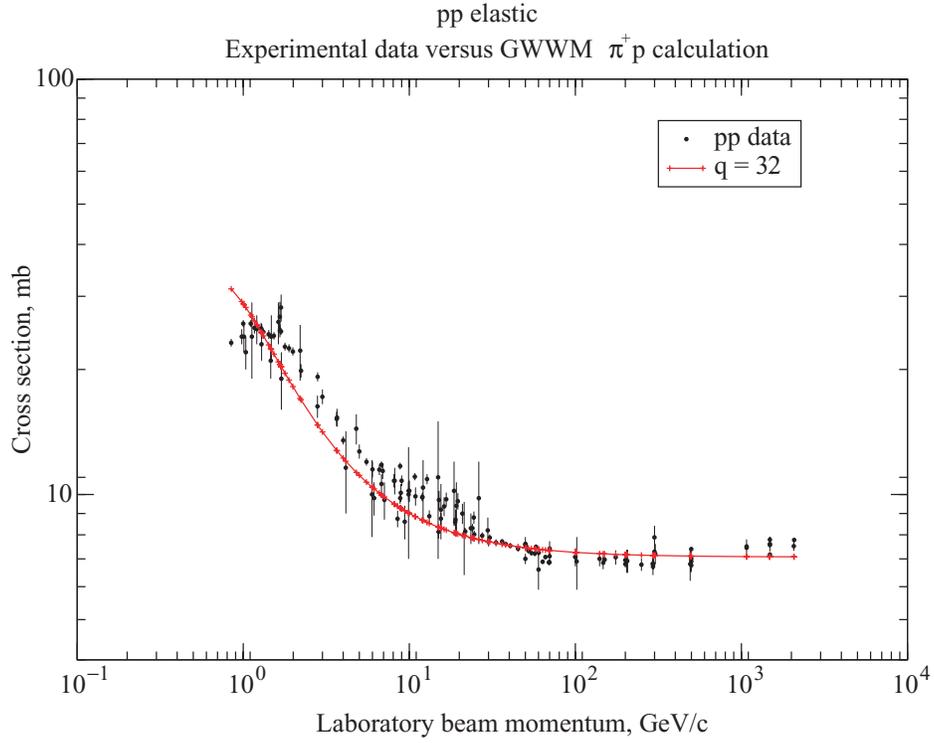


Figure 1. Predictions of the GWWM and data from review of particle physics (ref. 8) using only the transverse number spectrum ($NT(E)$).

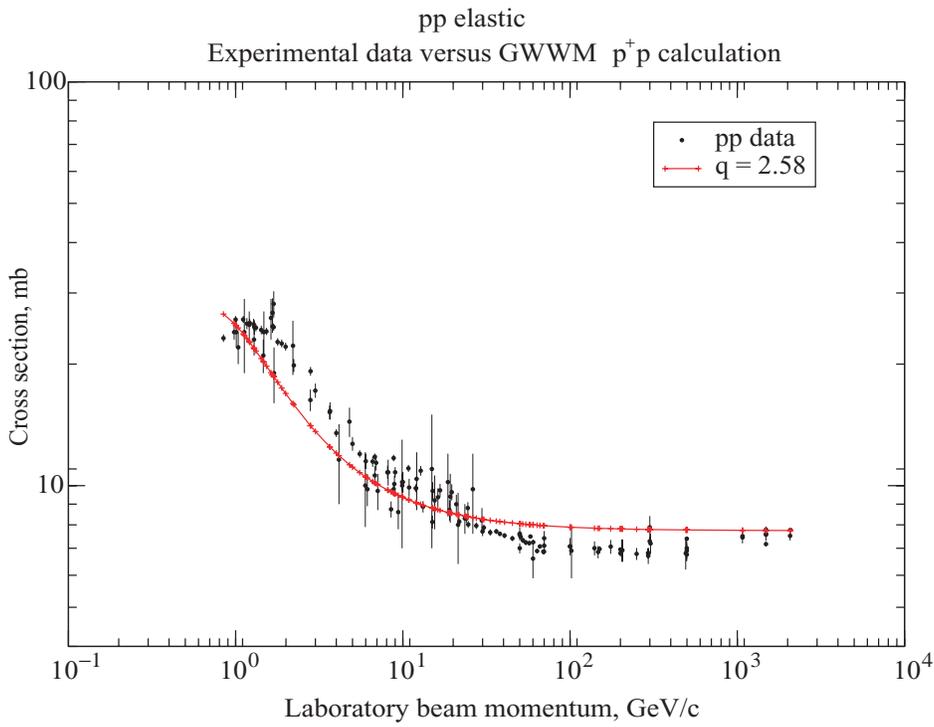


Figure 2. Predictions of the GWWM and data from review of particle physics (ref. 8) using both transverse and longitudinal number spectrums ($N_T(E) + N_L(E)$).

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